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On The Integrability Conditions of the F-Structure Satisfying $F^3 + F^2 + F = 0$

Abstract

The purpose of this paper is to study various properties of the structure equation $F^3 + F^2 + F = 0$. The integrability conditions have also been discussed.

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Introduction

Le V_n be a C^∞ difference manifold and F be a C^∞ (1,1) tensor defined on V_n such that (1. 1) $F^3 + F^2 + F = 0$.

We define the operators l and m on V_n by (1.2) $l = F^3$, $m = I - F^3$

Where I is the identity operator

From (1. 1) and (1.2) we have

$l + m = I,\ l^2 = l,\ m^2 = m,\ lm = ml = 0$ (1.3)

$Fl = lF = F,\ Fm = mF = 0$

Let

$(M_6 = \{m-F^3, m-F^2, m-F, m+F, m+F^2, m+F^3\}$ (1.4)

Theorem (1.1)

The set defined by (1.4) is a cyclic and non-simple group under product of operators

Proof

Using (1.1),(1.2), (1.3) and (1.4). We have the cayley table for M_6 (1.5)

	$m-F^3$	$m-F^2$	$m-F$	$m+F$	$m+F^2$	$m+F^3$
$m-F^3$	$m+F^3$	$m+F^2$	$m+F$	$m-F$	$m+F^2$	$m-F^3$
$m-F^2$	$m+F^2$	$m+F$	$m+F^3$	$m-F^3$	$m-F$	$m-F^2$
$m-F$	$m+F$	$m+F^3$	$m+F^2$	$m-F^2$	$m-F^3$	$m-F$
$m+F$	$m-F$	$m-F^3$	$m-F^2$	$m+F^2$	$m+F^3$	$m+F$
$m+F^2$	$m-F^2$	$m-F$	$m-F^3$	$m+F^3$	$m+F$	$m+F^2$
$m+F^3$	$m-F^3$	$m-F^2$	$m-F$	$m+F$	$m+F^2$	$m+F^3$

From the table (1.5) we observed that M_6 is closed under multiplication of operators. Associative property is obviously satisfied. $M+F^3$ acts as identity operator. Also

$(m-F^3)^{-1} = m-F^3$ (1.6)

$(m-F^2)^{-1} = m-F$

$(m+F)^{-1} = m+F^2$

$(m+F^3)^{-1} = m+F^3$

and M_6 is a cyclic group generated by $m-F$ or $m-F^2$. Since M_6 has a proper normal subgroups $\{m-F^3, m+F^3\}$, consequentlY M_6 is not a simple group.

Nijenhuis Tensor

Let N_m denotes the Nijenhuis tensor corresponding to the operator m, then

$N_m(X, Y) = [mX, mY] + m^2 [X, Y] - m [mX, Y] - m [X, m Y]$ (2.1)

Theorem (2.1) for the Nijenhuis tensor ${}^N{}_m$ defined by (2.1), we have

1. ${}^N{}_m(lX, lY) = m [lX, lY]$ (2.2)
2. ${}^N{}_m(lX, mY) = 0$
3. ${}^N{}_m(mX, lY) = 0$
4. ${}^N{}_m(mX, mY) = l[mX, mY]$

Proof

using (1.3) and (2.1), we have

${}^N{}_m(lX, lY) = [mlX, mlY] + m^2 [lX, lY] - m [mlX, lY] - m [lX, mlY]$ (2.3)

$= m[lX, lY]$

Proceeding similarly the other parts follow.

Integrability Conditions

Let l* and m* denote the tangent bundles associated with the complementary projection operators l and m respectively, then

Theorem (3.1)l* is integrable if and only if

${}^{N}{}_{m}(lX, lY) = 0$ (3.1)

Proof

l* is integrable if and only if

mX = 0 for X = lX

$\Longrightarrow$ (dm) (lX, lY) = 0, simplifying it

m[lX, lY] = 0 (3.2)

From (2.2) (i) and (3.2), we get (3.1)

Theorem (3.2) m* is integrable if and only if

${}^{N}{}_{m}(mX, mY) = 0$ (3.3)

Proof

m* is integrable if and only if

lX = 0 for X = mX

$\Longrightarrow$ (dl) (mX, mY) = 0, simplifying it

l[mX, mY] = 0 (3.4)

From 2.2) (iv) and (3.4), we get (3.3)

Theorem (3.3) The differentiable manifold V_n regarded as the sum of l* and m* is integrable if and only if

${}^{N}{}_{m}(X, Y) = 0$ (3.5)

Proof

using (1.3) and bilinearity of ${}^{N}{}_{m}$, we have

${}^{N}{}_{m}(X, Y) = {}^{N}{}_{m}(lX+mX, lY+ mY)$ (3.6)

$= {}^{N}{}_{m}(lX, lY) + {}^{N}{}_{m}(lX, mY) + {}^{N}{}_{m}(mX, lY) + {}^{N}{}_{m}(mX, mY)$

Using (2.2) (ii), (iii)

${}^{N}{}_{m}(X, Y) = {}^{N}{}_{m}(lX, lY) + {}^{N}{}_{m}(mX, mY)$ (3.7)

From theorems (3.1) and (3.2) and (3.7), V_n is integrable if and only if ${}^{N}{}_{m}(X, Y) = 0$

Aim of Study

The integrability condition of $F^3 + F^2 + F = 0$.

Conclusion

The given structure is integrable if and only if ${}^{N}{}_{m}(X, Y) = 0$.

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